Problem Set 13
Due: Wednesday, November 28

Work all of the following problems. A subset of the problems will be graded. Be sure to adhere to the expectations outlined in the *General Problem Set Guidelines Sheet*.

Unless otherwise stated, all problems can be found in the appropriate *Exercises* sections of the text (*Abstract Algebra* by D. Dummit and R. Foote, 3rd Edition).

- Section 14.2 # 3, 4, 7, 12, 13
- Let K be the splitting field of $f(x) = x^4 - x^2 + 1$ over $\mathbb{Q}$. Find the Galois group of K over $\mathbb{Q}$. Express every element of $Gal(K/\mathbb{Q})$ as a permutation of the roots of $f(x)$.
- Let K be a finite extension of $\mathbb{Q}$. Let $f(x) \in \mathbb{Q}[x]$.
  (a) Suppose $[K : \mathbb{Q}] = 5$. Assume that all roots of $f(x)$ are in K, but not all of the roots are in $\mathbb{Q}$. Prove that K is the splitting field of $f(x)$.
  (b) Give an explicit example to show that part (a) is false if $[K : \mathbb{Q}] = 4$.
  (c) Suppose K is a Galois extension of even degree greater than or equal to 4 over $\mathbb{Q}$. Assume that $f(x)$ is a separable and reducible polynomial of degree 4 such that all roots of $f(x)$ are in K, but none of them are in $\mathbb{Q}$. Prove that the Galois group of K over $\mathbb{Q}$ is not simple.

  Note: For Section 14.2 # 7, you may use the work presented in the Example on pages 577 – 581 of the textbook.