
Problem Set 4

Due: Wednesday, September 26

Work all of the following problems. A subset of the problems will be graded. Be sure to adhere to the expectations outlined in the *General Problem Set Guidelines Sheet*.

Unless otherwise stated, all problems can be found in the appropriate *Exercises* sections of the text (*Abstract Algebra* by D. Dummit and R. Foote, 3rd Edition).

- Section 7.6 # 3, 5 parts (b) and (c)
- Section 8.1 # 3, 4
- Find a generator for the ideal $(85, 1 + 13i)$ in $\mathbb{Z}[i]$.
- Let $F = \mathbb{Q}(\sqrt{-2})$ be the quadratic field with associated quadratic integer ring \mathcal{O} and field norm N as in Section 7.1. Prove that \mathcal{O} is a Euclidean Domain with respect to N . (*Hint*: Modify the proof for $\mathbb{Z}[i]$.)
- In class we proved that $I = (2, 1 + \sqrt{-5})$ is not a principal ideal of $\mathbb{Z}[\sqrt{-5}]$. Prove that $J = (3, 2 - \sqrt{-5})$ is also not principal yet IJ is a principal ideal in $\mathbb{Z}[\sqrt{-5}]$. You may assume the results of Section 7.4 # 12.