Study Guide - Final Exam

Date and Location: The Final Examination will be given on Tuesday, December 6 from 8:00 a.m. to 9:50 a.m in Pearce Hall, Room 123A.

Material Covered: You will be responsible for the material in Chapters 1 - 4 (excluding Young's Geometry in Section 1.3 and the discussion on congruence and area in Section 4.3) from the textbook.

Exam Aids: You will be permitted to use self-prepared notes. The notes must be hand-written and contained on one-side of an $8.5'' \times 11''$ piece of paper. You will be required to submit the piece of paper with your exam. **You will not be permitted to use any electronic technology or other aids during the exam. The only items you can use for the exam are a pen and/or pencil, eraser, your self-prepared notes, and a compass and straightedge.** Questions will be similar to those on Exams 1 & 2 and Problem Sets completed throughout the semester.

Summary of Key Points: Below is a summary of the key points that you are responsible for.

- History of geometry – materials from the textbook and class discussions. You will not be asked for specific locations or dates.
- Definitions and importance of: Axiomatic Method, primitive/undefined terms, axioms/postulates, theorems, etc.
- Models (concrete and abstract) and isomorphisms between models.
- Properties of Axiomatic Systems: consistency, absolute consistency, relative consistency, independence, completeness. You should know how to test for these properties as well as what the properties mean.
- Finite geometries.
- Euclid's *Elements*: you should be able to state Euclid's 5 postulates in modern English, be aware of some of the definitions, explain the importance of the 5th postulate, know some of the shortcomings of Euclid's axiomatic set.
- Playfair's Postulate.
- Hilbert's, Birkhoff's, and SMSG's Axiom Sets for Euclidean Geometry. (No need to memorize the full details, but know why each one is important and what makes each one unique. Know the names of the postulates and theorems we have used often, such as SAS Congruency or Pasch's Axiom, and in which axiomatic set they are used as a postulate or as a theorem.)
- Properties of Elliptic (spherical) Geometry.
- Properties of Hyperbolic Geometry – Poincaré disk and half-plane models.
- What Neutral Geometry is and how it differs from Euclidean Geometry.
- Congruence of line segments, angles, and polygons - definitions and basic facts.
- The Isosceles Triangle Theorem along with its converse and inverse statements.
- The Line Segment Perpendicular Bisector Theorem (Theorem 3.2.8).
- The definition of an exterior angle to a triangle and the Exterior Angle Theorem.

- Congruency conditions for triangles and quadrilaterals.
- The Triangle Inequality Theorem.
- The Hinge Theorem.
- The Alternate Interior Angle Theorem.
- The Converse of the Alternate Interior Angle Theorem and its equivalence to the Euclidean Parallel Postulate.
- The equivalence of Euclid's Fifth Postulate and the Euclidean Parallel Postulate.
- The Saccheri-Legendre Theorem and its two preliminary lemmas (Lemmas 3.5.2 and 3.5.3).
- Saccheri and Lambert quadrilaterals - definitions and facts.
- The relationship between rectangles, angle sums of triangles, and the Euclidean Parallel Postulate.
- Implications of the Euclidean Parallel Postulate stated in Section 4.2.
- Parallelograms, squares, and other special quadrilaterals - definitions and properties.
- The Median Concurrence Theorem and Corollary 4.2.8.
- Similarity of polygons - definition and conditions (AAA, SAS, SSS).
- The Basic Proportionality Theorem.
- Definitions related to circles: circle, radius, chord, diameter, secant, tangent, central angle, semicircle, minor arc, major arc, measure of an arc, inscribed angle, intercepted arc.
- The fact that in the Euclidean plane, a circle is uniquely determined by three distinct, non-collinear points.
- Basic facts about chords, diameters, and arcs.
- The Arc Addition Theorem.
- The Inscribed Angle Theorem.
- Angle theorems related to chords, secants, and tangents.
- Product theorems related to chord segments, secant segments, and tangent-secant segments.
- Special points related to triangles: centroid, circumcenter, orthocenter, incenter, excenter. You should know their basic properties and relationships.
- The Theorem of Menelaus.
- Ceva's Theorem.
- The Nine-Point Circle - you should know the special 9 points on this circle along with what the center and radius are. You should also know properties of the Nine-Point Circle for special triangles.
- Euclidean constructions - you should be able to complete the six constructions discussed in class and understand why they work, explain how to construct sums, differences, products, quotients, explain how to construct the number $\sqrt{x}$ for any rational number x, and have some understanding of the two classical construction problems discussed in class (constructing a cube with twice the volume of any cube, constructing a square having the same area as any given circle). **You will need a compass and straightedge for the final examination.**