Lab 2
Introduction to Limits and Basic Laws

1. Evaluate the following limits either graphically or numerically. Each answer should be of the form ∞, $-\infty$, or "does not exist".

   (a) $\lim_{x\to -3^+} \dfrac{x+2}{x+3}$

   (b) $\lim_{x\to -3^-} \dfrac{x+2}{x+3}$

   (c) $\lim_{x\to -3} \dfrac{x+2}{x+3}$

   (d) $\lim_{x\to 1} \dfrac{2-x}{(x-1)^2}$

   (e) $\lim_{x\to 3^+} \ln(x^2-9)$

   (f) $\lim_{x\to \pi^-} \cot(x)$

2. Let

$$f(x) = \begin{cases} 2-x, & \text{if } x < -1 \\ x, & \text{if } -1 \le x < 1 \\ (x-1)^2, & \text{if } x > 1 \end{cases}$$

   Find the following values, if they exist.

   (a) $\lim_{x\to -1^-} f(x)$

   (b) $\lim_{x\to -1^+} f(x)$

   (c) $\lim_{x\to -1} f(x)$

   (d) $f(-1)$

   (e) $\lim_{x\to 1^-} f(x)$

   (f) $\lim_{x\to 1^+} f(x)$

   (g) $\lim_{x\to 1} f(x)$

   (h) $f(1)$

3. Write the function $f(x) = \dfrac{|x-1|}{x-1}$ as a piece-wise defined function in order to compute the following limits.

   (a) $\lim_{x\to 1^+} \dfrac{|x-1|}{x-1}$

   (b) $\lim_{x\to 1^-} \dfrac{|x-1|}{x-1}$

(c) $\lim_{x\to 1} \dfrac{|x-1|}{x-1}$

4. Use Limit Laws to find the following limits.

   (a) $\lim_{x\to 1} \left(\dfrac{1+3x}{1+4x^2+3x^4} \right)^3$

   (b) $\lim_{t\to 4^-} \sqrt{16-t^2}$

   (c) $\lim_{t\to 4^+} \sqrt{16-t^2}$

5. Show that $\lim_{x\to 2}([x]-[-x])$ exists and is not equal to $f(2)$, where $[x]$ denotes the greatest integer function.

6. Assume that
$$\lim_{x\to 0} \frac{\sin x}{x} = 1.$$
Use this fact, some manipulation, and Limit Laws to find
$$\lim_{x\to 0} \frac{\cos x - 1}{x}.$$

7. Assume that $\lim_{x\to 0} \dfrac{f(x)}{x^2} = 5$. Use this to find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} \dfrac{f(x)}{x}$.