Lab 3
Limits and Continuity

1. Evaluate the following limits.

   (a) $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$

   (b) $\lim_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7}$

   (c) $\lim_{h\to 0} \dfrac{(2+h)^3-8}{h}$

   (d) $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$

   (e) $\lim_{u\to 0} \left(\dfrac{1}{u}-\dfrac{1}{u^2+u}\right)$

   (f) $\lim_{x\to 0} \left(\dfrac{1}{x\sqrt{x^2+1}}-\dfrac{1}{x}\right)$

   (g) $\lim_{x\to -6} \dfrac{2x+12}{|x+6|}$

   (h) $\lim_{u\to 0} \left(\dfrac{1}{u}-\dfrac{1}{|u|}\right)$

   (i) $\lim_{h\to 1} \dfrac{h^{1/3}-1}{h-1}$

   (j) $\lim_{x\to 0} \tan(e^x)$

2. Find an example of functions f and g such that neither $\lim_{x\to c} f(x)$ nor $\lim_{x\to c} g(x)$ exists but $\lim_{x\to c}[f(x)g(x)]$ exists.

3. Find all values of a that make the function

$$f(x) = \begin{cases} ax^2+ax, & \text{if } x<1 \\ -a^2x, & \text{if } x\geq 1 \end{cases}$$

   continuous for all real numbers x.

4. Find all values of a and b that make the function

$$f(x) = \begin{cases} ax+b, & \text{if } x\geq 0 \\ bx^2+ax, & \text{if } x<0 \end{cases}$$

   continuous for all real numbers x.

5. Consider the functions

$$f(x) = \begin{cases} x+1, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

and

$$g(x) = b^2x^2 + bx + b^2$$

where b is a constant. Find all values of b that make the composite function $g(f(x))$ continuous for all real numbers x.