Lab 4

Trigonometric Limits, Limits at Infinity, The Squeeze and Intermediate Value Theorems, and the Precise Definition of Limits

1. Use the Squeeze Theorem to find $\lim_{x\to 2}(x^2-4x+4)\sin\left(\dfrac{x-1}{x-2}\right)$.

2. Evaluate the following limits.

   (a) $\lim_{x\to 0}\dfrac{\sin(5x)\sin(2x)}{\sin(3x)\sin(4x)}$

   (b) $\lim_{x\to 0}\dfrac{\tan(2x)}{\sin(9x)}$

   (c) $\lim_{t\to 0}\dfrac{1-\cos(2t)}{\sin^2(3t)}$

   (d) $\lim_{t\to 0}\dfrac{\cos t-\cos^2 t}{t}$

   (e) $\lim_{h\to \frac{\pi}{2}}\dfrac{1-\cos 3h}{h}$

   (f) $\lim_{x\to -\infty}\dfrac{4x-3}{\sqrt{25x^2+4x}}$

   (g) $\lim_{x\to\infty}(\sqrt{9x^3+x}-x^{3/2})$

   (h) $\lim_{x\to\infty}(\ln(\sqrt{5x^2+2})-\ln(x))$

3. Does the Intermediate Value Theorem apply to the function $f(x)=\dfrac{1}{x-1}$ on the interval $[0,2]$?

4. Show that the following functions have at least one real solution.

   (a) $3x^3=1-\sin x$

   (b) $\tan x = 1-x$

   (c) $e^{-x^2}=x$

5. Suppose that f is a continuous function on $[1,5]$ and that the *only* solutions of the equation $f(x)=6$ are $x=1$ and $x=4$. If $f(2)=8$, explain why $f(3)>6$.

6. Use the formal definition of the limit to rigorously prove that $\lim_{x\to 1}(3x+2)=5$.

7. Use the formal definition of a limit to show that if $\lim_{x\to c}f(x)=L$ and $\lim_{x\to c}g(x)=K$, then
$$\lim_{x\to x}(f(x)+g(x))=L+K.$$
   Hint: You will need to use the Triangle Inequality $|a+b|\leq|a|+|b|$.