Lab 6
Higher Derivatives, Trigonometric Functions, and the Chain Rule

1. Compute the derivative for each of the following functions.
   (a) $y = \dfrac{t}{1 + \sec t}$
   (b) $y = \tan(t^{-3})$
   (c) $y = e^{4t - t^2}$
   (d) $y = \sin(2x)\cos^2 x$
   (e) $y = e^{(x^3+3x+4)^4}$
   (f) $y = (3e^{3x} + 3e^{-2})^4$
   (g) $y = \tan(\sqrt{1 + \csc x})$
   (h) $y = \cos(\cos(\cos(x)))$

2. Calculate y'' for each function y below.
   (a) $y = 12x^3 - 5x^2 + 3x$
   (b) $y = \sqrt{2x + 3}$
   (c) $y = \tan(x^2)$
   (d) $y = \sin^2(4x + 9)$

3. Find a general formula for $f^{(n)}(x)$ where $f(x) = (x + 2)^{-1}$.

4. Find a polynomial $f(x)$ that satisfies the equation
$$xf''(x) + f(x) = x^2.$$

5. Find $f'(2)$ if $f(g(x)) = e^{x^2}$, $g(1) = 2$, and $g'(1) = 4$.

6. Find an equation of the tangent line to the curve $f(x) = \csc x - \cot x$ at the point $x = \pi/4$.

7. A particle moves in a straight line with displacement $s(t)$, velocity $v(t)$, and acceleration $a(t)$.
   (a) Show that
$$a(t) = v(t)\frac{dv}{ds}.$$
   (b) Explain the difference between the meanings of $\dfrac{dv}{dt}$ and $\dfrac{dv}{ds}$.

8. Recall that a function f is *odd* if $f(-x) = -f(x)$ for all x and *even* if $f(-x) = f(x)$ for all x. Prove that if f is an odd function, then f' is an even function. Does a similar statement hold for even functions?