Lab 7
Exponential & Logarithmic Functions, Implicit Differentiation, Related Rates, and Linear Approximation

1. Compute the derivative for each of the following functions.

   (a) $f(x) = 7^{-2x}$

   (b) $g(x) = \tan^{-1}(\ln x)$

   (c) $f(\theta) = \ln(\sin \theta)$

   (d) $y = x^{\sinh x}$

   (e) $g(t) = \sinh(t^2)$

   (f) $g(x) = \tanh^{-1}(e^x)$

   (g) $y = xy^2 + 2x^2$

   (h) $y = \sin(x + y)$

2. Find all points on the graph of $3x^2 + 4y^2 + 3xy = 24$ where the tangent line is horizontal.

3. A hot air balloon rising straight up from a level field is tracked by a range finder 500 feet from the liftoff point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 radians per minute. How fast is the balloon rising at that moment.

4. Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep?

5. Find the linearization of the function $f(x) = \sqrt[3]{1 + 3x}$ at $a = 0$. Use this to give an approximate value for $\sqrt[3]{1.03}$ and determine if this is an underestimate or an overestimate (explain without using your calculator!).

6. Find the differential dy of the function $y = \cos \pi x$.

7. The Bonzo Pizza Company claims that its pizzas are circular with diameter 50 cm.

   (a) What is the area of the pizza?

   (b) Estimate the quantity of pizza lost or gained if the diameter is off by at most 1.2 cm.