Problem Set 15
Due: 3:00 p.m. on Wednesday, December 9

Instructions: Carefully read Section 12.2 of the textbook. Work all of the following problems. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets.* Submit your solutions in-class or to Dr. Cooper's mailbox in the Department of Mathematics.

Exercises: For this Problem Set, let F be a field and let A and B be elements of $M_{n \times n}(F)$.

1. (a) Prove that if A and B are similar then A and B have the same characteristic polynomial and the same minimal polynomial.

   (b) Show that if A and B have the same characteristic polynomial, then they need not be similar. Justify your work.

   (c) Prove or give a counter-example to the converse of part (a). Justify your answer.

2. We say that A is *nilpotent* if there is a natural number $q \geq 1$ such that $A^q = 0$.

   (a) Prove that the following conditions are equivalent:

      (i) A is nilpotent;
      (ii) the minimal polynomial of A is of the form x^m;
      (iii) the characteristic polynomial of A is x^n.

   (b) Prove that A is nilpotent if and only if $A^n = 0$.

3. Find the rational canonical form of the following matrix in $M_{3 \times 3}(\mathbb{R})$.

$$\begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 5 & 3 \end{pmatrix}$$