Problem Set 7
Due: 3:00 p.m. on Thursday, October 15

Instructions: Carefully read Sections 9.3, 9.4, and 9.5 of the textbook. Work all of the following problems. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. Submit your solutions in-class or to Dr. Cooper's mailbox in the Department of Mathematics.

Exercises:

1. (Dummit and Foote §9.3 #1) Let R be an integral domain with quotient field F and let $p(x)$ be a monic polynomial in $R[x]$. Assume that $p(x) = a(x)b(x)$ where $a(x)$ and $b(x)$ are monic polynomials in $F[x]$ of smaller degree than $p(x)$. Prove that if $a(x) \notin R[x]$ then R is not a Unique Factorization Domain. Deduce that $\mathbb{Z}[2\sqrt{2}]$ is not a U.F.D.

2. (Dummit and Foote §9.4 #7) Prove that $\mathbb{R}[x]/(x^2+1)$ is a field which is isomorphic to the complex numbers.

3. (Dummit and Foote §9.5 #1) Let F be a field and let $f(x)$ be a non-constant polynomial in $F[x]$. Describe the nilradical of $F[x]/(f(x))$ in terms of the factorization of $f(x)$.

4. Prove that the rings $\mathbb{Q}[x,y]/(y^2-x^3)$ and $\mathbb{Q}[t^2,t^3]$ (where x, y and t are indeterminates) are isomorphic. (*Hint:* Consider the ring homomorphism $\phi : \mathbb{Q}[x,y] \to \mathbb{Q}[t^2,t^3]$ mapping $f(x,y)$ to $f(t^2,t^3)$ and use the Division Algorithm to determine the kernel of ϕ.)