Math 824: Topics in Commutative Algebra
The Power of Monomial Ideals
Course Information Sheet and Syllabus[1]
Fall 2016

Instructor:

Dr. Susan Cooper

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Office Hours: Mondays & Wednesdays & Fridays 10:00 a.m. – 10:50 a.m.; or by appointment
Correspondence: The most reliable way to contact me is via email.

Class Times and Location: MWF 9:00 a.m. – 9:50 a.m., Ehly Hall – Room 220;
T 9:00 a.m. - 9:50 a.m., Minard Hall – Room 404 (until the end of September)

Prerequisites: Math 720

Credit Hours: 3

Course Web-Page: We will use *Blackboard* which can be found at https://bb.ndsu.nodak.edu/.

Textbook: Course material will come from a variety of sources, including research papers. Some useful books include:

- *Monomial Algebras*, Second Edition, by Rafael H. Villarreal
- *Combinatorial Commutative Algebra* by Ezra Miller and Bernd Sturmfels
- *Cohen-Macaulay Rings*, Revised Edition, by Winfried Bruns and Jürgen Herzog

Bulletin Description: Topics vary each time the course is offered and may include: dimension theory, integral dependence, factorization, regular rings, Cohen-Macaulay rings, Gorenstein rings. May be repeated for credit with change in subtopic.

Course Introduction and Objectives: Monomial ideals have played a central role in numerous problems in Commutative Algebra and Algebraic Geometry. Not only are they interesting ideals to work with, but many complicated problems are better understood by reducing to the monomial case. For example, monomial ideals are important for characterizing Hilbert functions and graded Betti numbers. Hilbert functions and graded Betti numbers were introduced by David Hilbert in his investigations of how the dimensions of the space of invariants of degree d varies with d. To record this data, if I is a homogeneous ideal in the polynomial ring then we incorporate the degree-by-degree dimensions of I in a sequence called the Hilbert function. Related to the Hilbert function are the graded Betti numbers which are invariants obtained by looking at the relations on the generators of I, and the relations on these relations (called the syzygies), etc. Hilbert functions and Betti numbers can be exploited to obtain both algebraic and geometric information. Monomial ideals give important examples which have extremal values of these invariants. Another example of the power of monomial ideals comes from Combinatorial Commutative Algebra in which properties of monomial ideals have proven to be useful in studying combinatorial problems.

It is intended that the precise topics covered will be driven by the interests and backgrounds of the course participants. Time permitting, we will explore topics such as Macaulay's Theorem (which characterizes Hilbert functions of homogeneous ideals via special monomial ideals), lifting monomial ideals to obtain the Hilbert functions of finite sets of projective points, Stanley-Reisner rings and edge ideals. In addition to being exposed to various research topics and building a library of examples, participants will strengthen writing and presentation skills which are crucial in an academic setting.

[1] *The details stated in this course syllabus are subject to change at the discretion of the instructor. Announcements concerning all (if any) changes will be made in a timely fashion.*

Problem Sets: Mathematics is not a spectator sport. The best way to learn mathematics is by doing mathematics. Homework will be assigned throughout the course. A subset of the solutions will be graded based on correctness, clarity, and style/creativity. The Problem Sets are intended to gauge your understanding of the material and all feedback is meant to improve your abilities and communication. In addition to written submissions, students will be expected to present solutions and contribute to the class discussions related to the Problem Sets (to be reflected in the Problem Sets grade).

Course Projects: Each student will be required to review literature (either from a research paper or a textbook chapter) on a topic related to monomial ideals. Each student will obtain permission to work on the chosen topic, and each student will explore a different topic. The project will consist of: a written 3–5 page paper summarizing main results, definitions, and examples; a 50-minute class presentation; and a related homework problem to be assigned to classmates. Feedback from the entire class will be given on the presentations. Full details for the project assignment requirements and topics will be given later in the semester, but important dates to keep in mind for the project are:

Task	Due Date
Topic Proposals	October 3 (9:00 a.m.)
Papers & Proposed Problem	December 2 (5:00 p.m.)
Presentations	December 5, 7, 9 (in class) and December 12 (Final Exam Period)
Final Problem Set	December 15 (9:00 a.m.)

Missed/Late Work Policies: A late submission of a Problem Set will only be granted for an unavoidable, documented circumstance as described below:

Circumstance	Required Documentation
illness or other medical situation	official note from clinic, hospital, doctor, nurse, or other health care provider
military service	official military activation orders
funeral or other family emergency	official documentation from newspaper, funeral, or medical official
sports or other official NDSU activity	official documentation from NDSU athletics or activity's faculty adviser

No extension will be given for a course project paper, proposed problem, or related final Problem Set. A project presentation will only be rescheduled for an unavoidable, documented circumstance.

Course Grades: Final course grades will be determined as follows:

Task	Percentage of Grade
Problem Sets	45%
Course Project Paper	25%
Course Project Presentation	25%
Course Project Problem	5%

Percentage Grade	Grade Earned
90% – 100%	A
80% – 89%	B
70% – 79%	C
60% – 69%	D
0% – 59%	F

Attendance and Participation: Your understanding of the course material will be supported by regular attendance and engagement in class meetings. According to NDSU Policy 333: Class Attendance Policy and Procedure (see `www.ndsu.edu/fileadmin/policy/333.pdf`), attendance in classes is expected. You are expected to be an active participant; this includes participating fully in classroom activities and critically thinking about the mathematics during and outside of class. *In order for this class to be successful, it is imperative that you commit to coming to class regularly, that you commit to coming to class prepared, and that you commit to participating in class!* Attendance will not be taken, but Problem Set grades will include your participation in solution presentations. Veterans and student service members with special circumstances or who are activated are encouraged to notify the instructor as soon as possible and are encouraged to provide Activation Orders.

Tentative Course Schedule and Calendar of Events: Approximately, Math 824 will meet 4 times a week until the end of September and 3 times a week in October. November will be dedicated to project preparation. Presentations will be given in the last week of classes and the final exam week.

Dates(s)	Topic/Event
August 23	First Class Meeting
September 5	Labor Day (No Class)
October 3	Course Project Topics Proposal Due
November 11	Veterans Day (No Class)
November 24–November 25	Thanksgiving (No Classes)
December 2 (5:00 p.m.)	Course Project Papers & Proposed Problems Due
December 5–December 9	Course Project Presentations (In Class)
December 9	Last Day of Classes
December 12 (8:00–10:00 a.m.)	Course Project Presentations (Final Examination Period)
December 15 (9:00 a.m.)	Final Problem Set Due

Other Resources: Please note that it is your responsibility to prepare clear and thorough notes – these will provide you with clarifying examples and reasoning. In addition, it is your responsibility to check your Blackboard and NDSU email accounts regularly for class announcements.

Special Concerns: Any students with disabilities or other special needs, who need special accommodations in this course, are invited to share these concerns or requests with the instructor and contact the Disability Services Office (231-8463; `http://www.ndsu.edu/disabilityservices/`) as soon as possible.

Academic Honesty: The academic community is operated on the basis of honesty, integrity, and fair play. NDSU Policy 335: Code of Academic Responsibility and Conduct applies to cases in which cheating, plagiarism, or other academic misconduct have occurred in an instructional context. Students found guilty of academic misconduct are subject to penalties, up to and possibly including suspension and/or expulsion. Student academic misconduct records are maintained by the Office of Registration and Records (`https://www.ndsu.edu/registrar/`). Informational resources about academic honesty for students and instructional staff members can be found at `www.ndsu.edu/academichonesty`.

Any student found guilty of academic dishonesty in this course will receive a grade of 0 for the task in question. In addition, every such student will be reported to the Chair of Mathematics, the Dean of the College of Science and Mathematics, the Provost, and the Registrar. Students found guilty of a second offense of academic dishonesty in this course will receive a course grade of F.

Classroom Atmosphere and Courtesy: A part of learning is making mistakes. We want to establish a classroom atmosphere where the inevitable false starts and mistakes become an opportunity to improve – not an opportunity for embarrassment. Please be constructive and polite in questioning your colleagues in class. In addition, cellular telephones, pagers, and other similar devices are not to be used and are to be turned off or set to vibrate-mode during class-time.

Expectations and General Tips for Success: I ask that you have a well-defined sense of professionalism, that you always put forth your best effort, and that you develop a sense of responsibility to your educational community. Also, I ask that you exhibit a persistent desire to learn and that you are positive, open, and responsive to feedback. In return I will provide you with significant support. Relax and have fun with the course – I look forward to working with you!