Math 824 – Topics in Commutative Algebra
The Power of Monomial Ideals
Fall 2016

3 Credits

Instructor: Dr. Susan Cooper

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Course Description: Monomial ideals have played a central role in numerous problems in Commutative Algebra and Algebraic Geometry. Not only are they interesting ideals to work with, but many complicated problems are better understood by reducing to the monomial case. For example, monomial ideals are important for characterizing Hilbert functions and graded Betti numbers. Hilbert functions and graded Betti numbers were introduced by David Hilbert in his investigations of how the dimensions of the space of invariants of degree d varies with d. To record this data, if I is a homogeneous ideal in the polynomial ring then we incorporate the degree-by-degree dimensions of I in a sequence called the Hilbert function. Related to the Hilbert function are the graded Betti numbers which are invariants obtained by looking at the relations on the generators of I, and the relations on these relations (called the syzygies), etc. Hilbert functions and Betti numbers can be exploited to obtain both algebraic and geometric information. Monomial ideals give important examples which have extremal values of these invariants. Another example of the power of monomial ideals comes from Combinatorial Commutative Algebra in which properties of monomial ideals have proven to be useful in studying combinatorial problems.

In this course we will explore topics such as Macaulay's Theorem (which characterizes Hilbert functions of homogeneous ideals via special monomial ideals), lifting monomial ideals to obtain the Hilbert function of a finite set of projective points, Stanley-Reisner rings and edge ideals. In addition to assignments, students will be required to complete a project which will involve reading a research paper and giving a presentation.

Prerequisite: Math 720

Resources: Course material will come from a variety of sources, including research papers. Some useful books include:

- *Monomial Algebras*, Second Edition, by Rafael H. Villarreal (official textbook)
- *Combinatorial Commutative Algebra* by Ezra Miller and Bernd Sturmfels
- *Cohen Macaulay Rings*, Revised Edition, by Winfried Bruns and Jürgen Herzog

Scheduling: Math 824 will meet 4 times a week until the end of September and 3 times a week in October. November will be dedicated to project preparation. Presentations will be given in the last week of classes and the final exam week.

For Further Information: Please contact Dr. Cooper by email.