Problem Set 10
Due: Wednesday, April 15

1. Let $p(x) = x^3 + 9x + 6$.
   (a) Show that $p(x)$ is irreducible in $\mathbb{Q}[x]$.
   (b) Let θ be a root of $p(x)$. Find the inverse of $1 + \theta$ in $\mathbb{Q}(\theta)$.
2. Show that $\theta = \sqrt{1 + 3\sqrt{3}}$ is algebraic over $\mathbb{Q}$ of degree 4.
3. Prove that $f(x) = x^3 - 3$ is irreducible over the field $F = \mathbb{Q}(i)$.
4. Let F be a field.
   (a) Prove that an element α is algebraic over F if and only if the simple extension $F \subseteq F(\alpha)$ is finite.
   (b) Prove that if the field extension $F \subseteq K$ is finite, then it is algebraic.
   (c) Assume that $[F(\alpha) : F]$ is odd. Prove that α^2 is algebraic of finite degree over F and that $F(\alpha) = F(\alpha^2)$.