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## Problem Set 12

### Due: Wednesday, April 29

1. For each polynomial  $f \in \mathbb{Q}[x]$  below, find a splitting field  $L$  for  $f$  over  $\mathbb{Q}$  and  $[L : \mathbb{Q}]$ .
  - (a)  $f(x) = x^4 + 2$
  - (b)  $f(x) = x^4 + x^2 + 1$
  - (c)  $f(x) = x^6 - 4$
  
2. Let  $K$  be a field.
  - (a) Prove that  $\text{char}(K) = 0$  if and only if there is a homomorphism of fields  $\mathbb{Q} \rightarrow K$ .
  - (b) Prove that  $\text{char}(K) = p > 0$  if and only if there is a homomorphism of fields  $\mathbb{Z}/(p) \rightarrow K$ .