Problem Set 2

Due: 9:00 a.m. on Wednesday, January 27

Instructions: Carefully read Sections 1.1 and 1.2 of the textbook. Graduate students should submit solutions to all of the following problems and undergraduate students should submit solutions to only those marked with a "U". All students may submit solutions to the bonus problem. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. Submit your solutions in-class or to Dr. Cooper's mailbox in the Department of Mathematics.

Exercises: From pages 47–59 of the textbook.

1U. Let a, b and c be integers.

   (i) Use the definition of divisibility to prove that if $a|b$ and $a|c$, then $a|(b+c)$ and $a|(b-c)$.

   (ii) Prove that if $a \neq 0$, then $\gcd(a, b) = \gcd(a, a+b)$. (*Hint:* Show that $\gcd(a, b) \leq \gcd(a, a+b)$ and $\gcd(a, b) \geq \gcd(a, a+b)$.)

2U. Let a and b be positive integers. In class we proved that there exist integers q and r such that $a = bq + r$ and $0 \leq r < b$. Complete the proof of Euclid's Division Lemma by showing that the integers q and r are unique.

3U. Use the Euclidean Algorithm to find the greatest common divisor of 16261 and 85652. Also, use the Extended Euclidean Algorithm to find integers u and v such that

$$16261u + 85652v = \gcd(16261, 85652).$$

4U. Section 1.2 #1.11, page 50

5U. Section 1.3 #1.15, page 51

6. Section 1.3 #1.27(a), page 53

Bonus. Section 1.1 #1.4(b), pages 48–49