Problem Set 9
Due: 9:00 a.m. on Wednesday, March 23

Instructions: Carefully read Sections 3.4 and 3.5 of the textbook. Submit your solutions to the following problems. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. Submit your solutions in-class or to Dr. Cooper's mailbox in the Department of Mathematics.

Exercises: From pages 180–191 of the textbook.

1. Section 3.4 #3.14 parts (bi), (c) and (d), pages 183–184
2. Section 3.4 #3.15 parts (b) and (c), page 184
3. Section 3.4 #3.17(a), page 184
4. Section 3.5 #3.22(b), page 186
5. A *Mersenne prime* is a prime integer of the form $2^n - 1$ for some integer n. Mersenne primes are named after the French monk Marin Mersenne (1588–1648). It is still unknown whether there are infinitely many Mersenne primes. In Fall 2008, a research team at UCLA announced the discovery of a 13 million digit prime number - it is a Mersenne prime with $n = 43,112,609$. You can find a list of Mersenne primes at `www.mersenne.org/prime.htm`.
   (a) Factor each of the numbers $2^n - 1$ for $n = 2, 3, \ldots, 10$. Which ones are Mersenne primes?
   (b) Let p be a prime. Must $2^p - 1$ also be a prime integer? Either prove your answer or give a counter-example.
   (c) Prove that if $2^n - 1$ is a Mersenne prime, then n must also be a prime integer. *Hint: Note that $x^{cd} - 1 = (x^c)^d - 1^d$ for any integer x and positive integers c and d and recall the factorization of $x^n - y^n$ for any integers x, y and $n \geq 1$.*

Note: You may use Maxima for tedious computations. If you do so, then please still show sufficient work. The following commands may be helpful:

- to find $a \pmod{n}$ type the command $\text{mod}(a, n)$;
- to find the greatest common divisor of two positive integers a and b type the command $\gcd(a, b)$;
- to find the prime factorization of a positive integer n type the command $\text{factor}(n)$.