

Habits of Mind #5

Triangular Numbers

Carl Friedrich Gauss is one of the greatest mathematicians of all times. Legend has it that when he was in the third grade his teacher told his class to add the whole numbers from 1 through 100 and “I don’t want to see anyone’s eyes until you are finished.” Instantly, Carl raised his hand. After a scolding from the teacher for not following the rules, Carl said, “But I have the answer, it is 5050.” How did Carl get the answer so quickly?

Triangular numbers are numbers that you obtain by adding consecutive whole numbers starting with 1. For example, 1, 3, and 6 are triangular numbers as the following computation shows:

$$\begin{aligned}1 &= 1 \\3 &= 1 + 2 \\6 &= 1 + 2 + 3 \\&\dots\dots\end{aligned}$$

- (1) Continue the above list until you have the list of the first 15 triangular numbers.
- (2) Draw a circle. Place 8 dots fairly evenly around the circle. Connect each pair of dots by a line segment. (We call these segments **chords**.) How many distinct chords do you get? (Careful, if you used only 4 dots, then you would get 6 chords.)
- (3) Now suppose you have 15 students in your class and you want to pick two students to be “Class Leaders” for the day. You wonder, “How many different pairings are possible?” (“Tom and Mary K” and “Tom and Liz” would be two different pairings but “Liz and Tom” would be the same pair as “Tom and Liz”.)
- (4) Is there a relationship between the first three parts of this problem? If so, what is the relationship?
- (5) Suppose that we wanted to know the 50th triangular number. It would be tedious to list all the triangular numbers up to that point. Is there a formula we can use to find the 50th triangular number? I.e., given a whole number “ n ”, what is the n th triangular number?
- (6) What is the explanation as to how young Carl may have figured out the sum of the first 100 whole numbers? Can you generalize that explanation to explain why the formula works?

All explanations should be complete and enable the rest of us to understand the process.

Due Date: At the beginning of class on Wednesday, October 6.