Habits of Mind #7

Up or Down?

Dr. C likes card tricks, but we suspect they are often math problems in disguise. Today she showed us 100 cards numbered $1, 2, 3, \ldots, 99, 100$. To set up her card trick, she placed all the cards **face down** in numerical order $(1, 2, 3, \ldots, 100)$. For the first stage of the card trick, Dr. C flips each card face up. Next, she starts with card #2 and flips every 2nd card (i.e., she flips cards $\#2, \#4, \#6, \ldots, \#98, \#100$). For the third stage of the trick, she flips every 3rd card starting with card #3 (i.e., she flips cards $\#3, \#6, \#9, \#12, \ldots$). Observe that at the beginning of this third stage card #3 was face up, so after flipping card #3 will be face down. But card #6 was face down, so once it is flipped card #6 will be face up at the end of the third stage. For the 4th stage of the trick, Dr. C starts with card #4 and flips every 4th card.

Dr. C says she will continue the trick according to the same pattern. That is, for the nth stage of the trick, she starts at card #n and flips every nth card. Thus, when Dr. C gets to the last stage (the 100th stage) she flips card #100.

Dr. C's challenge for you is to determine the answers to the following questions:

(1) Which cards will be **face up** at the end of the trick? Explain *why* these particular cards are the ones face up at the end of the trick.

(2) If Dr. C had started with 1000 cards, which cards would be **face up** at the end of the trick?

Due Date: At the beginning of class on Wednesday, October 20.