

Perron, Frobenius and Big 12 Soccer

Math 314—002 Application Mini-Project #3

Due: Thursday, November 13

Goal: In this project you will learn about an application of the Perron-Frobenius Theorem to mathematical ranking systems (of sports teams, internet sites, etc.).

Instructions: *This project is to be done in groups of two or three students, as assigned by the instructor.* Groups of size four or more are not permitted. Each group will turn in one solution to the tasks described below, and each member of the group will receive the same grade on the project. You should be careful to understand each part of the project – related questions may appear on exams.

Submission Guidelines: Please use the following guidelines when preparing your project for submission:

1. Include a cover page on which each member of the group *signs* their full name. Also, in the top right hand corner of each page submitted, write the names (first and last, written legibly) of each group member. You should not include student ID or social security numbers.
2. Only use one side of each sheet of paper and staple the pages together.
3. Presentation will be considered when grading this project. You should take care to write legibly and hand in full sheets of paper with no fringe, tears, etc. You should also clearly label each problem and submit these in order.
4. Give justification (in complete sentences!) for your answers.
5. Be academically honest. This means, for example, providing a list of sources other than the textbook (if any) that you used to do the assignment; stating clearly that you're copying or mimicking an example from the book in order to do the assignment (if appropriate).
6. The project is due at the beginning of class. Under certain circumstances, late submissions may be accepted, but they will be penalized.

Project Tasks: Start by reading about the Perron-Frobenius theorem in Section 4.6 of your text. Then read the material on pages 353–355 which discusses the type of application this mini-project concerns.

1. Use the idea described in your reading to rank the 11 Women's Soccer teams in the Big 12 (Kansas State does not field a team) based on the outcome of the 2007 Big 12 Conference games. You can find the results of the games by following the links at the official Big 12 conference site:

<http://www.big12sports.com>

(Be sure to use data from the 2007-2008 season.)

A few more details:

- Ignore all non-conference games.
- On at least one occasion, two Big 12 teams played each other more than once, but only one game counted as a Big 12 Conference game. For the purposes of this project, ignore these “non-conference” games too.
- Ties can happen in soccer. For this part of the project, treat a tie as if the two teams never played each other.
- Be sure to demonstrate that the 11×11 matrix you get satisfies the hypotheses of the Perron-Frobenius theorem, so that a unique Perron eigenvector is guaranteed to exist.

2. Now redo the previous part, but this time treat ties differently. Put a $1/2$ in the appropriate positions of the matrix to represent each tie game. Once again, demonstrate that the hypotheses of the Perron-Frobenius theorem are met, and find the Perron eigenvector of the resulting matrix to rank the teams.
3. Explain why the approach concerning ties in (2) results in a reasonable ranking system. I am looking for some explanation similar to that on the top half of page 354.
4. The rankings you get in parts (1) and (2) should be a little different. Which one, in your opinion, is more reasonable? (I am not looking for anything rigorous here, just an opinion based on the data.)
5. What if we try to use the same system to rank the Big 12 football teams for this season based on the results through the games played on October 25th? (Again, ignore all non-conference games.) Does the Perron-Frobenius Theorem apply to this situation? Why or why not? You don't necessarily need to compute powers of the appropriate matrix (or even enter it in the computer) to answer this question.