Math 918–001: Topics in Algebra
The Power of Monomial Ideals
Spring 2010

Instructor: Dr. Susan Cooper

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Office Hours: Tuesdays 11:00 a.m. – 12:00 p.m.; Wednesdays 9:00 a.m. – 10:00 a.m.
Appointments: I am happy to make appointments when you are unable to attend office hours.
Correspondence: The most reliable way to reach me is via email.

Class Times and Location: TR 9:30 a.m. – 10:45 a.m., Avery Hall – Room 345.

Prerequisites: It will be assumed that the student is familiar with the basics of commutative algebra and algebraic geometry.

Course Web Page: `http://www.math.unl.edu/∼scooper4/math918.html`.

References: There will be no official text for this course. Resources that I will often refer to include:

- *Combinatorial Commutative Algebra* by E. Miller and B. Sturmfels;
- *Cohen Macaulay Rings* by W. Bruns and J. Herzog;
- *Commutative Algebra with a View Towards Algebraic Geometry* by D. Eisenbud;
- *Ideals, Varieties, and Algorithms* by D. Cox, J. Little, and D. O'Shea.

Other significant materials that will be used will be distributed to the class.

Course Description: Monomial ideals have played a central role in numerous problems in commutative algebra and algebraic geometry. Not only are they interesting ideals to work with, but many complicated problems are better understood by reducing to the monomial case. For example, monomial ideals are important for characterizing Hilbert functions and graded Betti numbers. Hilbert functions and graded Betti numbers were introduced by David Hilbert in his investigations of how the dimensions of the space of invariants of degree d varies with d. To record this data, if I is a homogeneous ideal in the polynomial ring then we incorporate the degree-by-degree dimensions of I in a sequence called the Hilbert function. Related to the Hilbert function are the graded Betti numbers which are invariants obtained by looking at the relations on the generators of I, and the relations on these relations (called the syzygies), etc. Hilbert functions and Betti numbers can be exploited to obtain both algebraic and geometric information. Monomial ideals give important examples which can be proved to have extremal values of these invariants.

The course will include an in-depth study of monomial ideals in general and will showcase a variety of special monomial ideals such as square-free monomial ideals. We will explore topics such as Gröbner bases and lifting monomial ideals. Throughout the course our attention will be focused on applying the theory of monomial ideals to understand a number of famous theorems such as Macaulay's Theorem (which characterizes Hilbert functions of homogeneous ideals) and popular conjectures such as the Eisenbud-Green-Harris Conjecture. Many situations will require us to draw on combinatorial tools. If there is enough interest and time, we may also investigate applications of monomial ideals to some problems arising from graph theory. The course is aimed at exposing students to several interesting research problems.

Group Presentations: In groups of two or three members, students will be required to review literature (either from a research paper or a text book chapter) on a topic related to monomial ideals. Groups will be required to obtain permission to work on the chosen topics, and each group will explore a different topic. Throughout the semester groups will spend one or two class periods presenting their findings. In addition, each group will be required to submit two related problems (with solutions) to be included in the course problem sets. Feedback from the entire class will be given on the presentations. All participants in a group will get the same grade on the presentation and problems, so it is important that each person in the group participate fully and equally. The group should consider meeting with me to discuss topics, references, presentation techniques, and the problems to be submitted.

Problem Sets: Problem sets will be assigned throughout the course. I will assign 3–4 short sets that will be used to gauge your progress with the material. Moreover, there will be 1–2 sets showcasing material based on the group presentations. Submitted work should be well-organized and clearly written.

Exams: There will be no tests and no final examination in this course.

Course Grades: Course grades will be based upon my assessment of your participation and level of engagement with the material during the semester. Roughly, grades for the course will be computed as follows:

Problem Sets	20%
Presentation	20%
Presentation Problems	15%
Attendance	20%
Participation	25%

Your participation is very important and strongly encouraged; please be an active participant and considerate of others during classroom discussions. My assessments will highly depend on the learning and growth I see in your work throughout the semester. Should you have any concerns about your progress, please do not hesitate to discuss these with me at any time.

Mathematics Department Grading Policy: Students who believe their academic evaluation has been prejudiced or capricious have recourse for appeals to (in order) the instructor, the department chair, the departmental appeals committee, and the college appeals committee.

Students with Disabilities: Students with disabilities are encouraged to contact me for a confidential discussion of their individual needs for academic accommodation. It is the policy of the University of Nebraska - Lincoln to provide flexible and individualized accommodation to students with documented disabilities that may affect their ability to fully participate in course activities or to meet course requirements. To receive accommodation services, students must be registered with the Services for Students with Disabilities (SSD) office, 132 Canfield Administration, 402-472-3787 voice or TTY.