Using Math to Understand Our World
Project 2
Using Body Temperature to Estimate Time Since Death

(A Solution to the *John Boddy Murder Mystery*)

Note: Answers to questions in boldface must be in your final report.

Part I: A Little History and One Method

Though there were some earlier studies on the temperature decrease in dead bodies, the first real scientific paper on the subject came out in 1868 and was written by one Harry Rainy, Professor of Medical Jurisprudence at the University of Glasgow. Rainy measured rectal temperatures of 100 patients who had died at the Glasgow Royal Infirmary. He took four or five such readings during the course of the first 2.5 days after death and calculated the rate of cooling per hour at different times (remember that this should be higher if the difference between the body's temperature and the room temperature is large). He probably really needed to take more measurements to get a good handle on how a body cools after death, but this was a start. Rainy discovered, as had others before him, that there is a "plateau" period after death where the body does not cool at all and the body temperature may even rise a bit. Rainy had the idea of applying *Newton's Law of Cooling*, but stated that "Bodies recently dead are not found to cool in conformity with this law." Rainy suggested that upon finding a dead body, the police surgeon take two temperature measurements separated by an hour, or, if possible, two or three hours. He goes on to say that "having obtained these data, we cannot exactly calculate the period which has elapsed since death, but we can almost always determine a minimum and maximum of time within which that period will be included." That is, we can compute two numbers t_{min} and t_{max}, and the time since death will very likely lie between these two numbers. Rainy suggested using *Newton's Law* to compute the minimum time, t_{min}. About the maximum he says "It is more difficult to fix a maximum time with precision, but a careful comparison of recorded cases will show that in all cases in which the temperature of the rectum is found to be below 85°F the time elapsed since death has been less than the minimum multiplied by 1.5." In other words, from looking at all the data he collected from dead bodies at the Glasgow Royal Infirmary, he noticed that the actual time since death was always less than $1.5 \times t_{min}$, provided t_{min} was computed by using data from the period when the body temperature was below 85°F.

Compute t_{min} and t_{max} for a body whose temperature was $80°$F when found and $76°$F 2 hours later. Assume the body is lying in a $60°$F room near the infirmary (remember that normal rectal temperature is $99.6°$F).

Since then, many people worked to refine Rainy's ideas. There are so many variables involved that even today there is not a perfect formula for computing the time since death. Some of the variables researchers have tried to take into account are: how much fat the body has; how thick the body is; how heavily dressed the body is; how humid the surrounding air is (in humid air, bodies cool more slowly); how much air circulation there is in the environment (bodies exposed to wind cool more quickly); and so on. Of course, in real life it is impossible for the coroner to know the values of all of those variables. It is especially impossible to know how all of the environmental variables may have changed since the time of death. Perhaps the temperature, humidity and wind velocity have increased or decreased during the time that the body was lying there.

One of the more useful algorithms for estimating time since death was invented by Professor T. K. Marshall of Belfast and his colleague F. E. Hoare in 1962. From their data they graphed the general shape of the curve which describes the temperature of a cooling body, which is shown below. Here $T_r(t)$ is the rectal temperature at time t and T_a is the temperature of the surroundings (also called the "ambient" temperature). **Why is there a horizontal line at $T = T_a$? Why does "$T_r - T_a$" appear in the middle of the picture? To what is it referring? Would it be appropriate to put this label anywhere else in the picture? Explain. Why does it say $T_0 - T_a$ on the vertical axis? Would it be appropriate to put this label anywhere else on the picture? Explain**. The temperature curve if the body cooled according to Newton's Law is also shown on this graph. **Identify which graph is which.**

Notice that, after a little time, these two graphs agree. Marshall and Hoare decided to search for a function whose graph looks like the one in Figure 1. (Mathematicians do this a lot- they start with a graph and work backwards to find a matching, or nearly matching, function.) Then this function can be used to compute the time since death in many different situations. In this case, finding a matching function is a little trickier than when we used *Newton's Law of Cooling*. Marshall and Hoare noticed that if they added together *two* exponential terms, one obtained from Newton's Law of Cooling and the other a "correction factor" to allow for the plateau, they could get a curve like the one in Figure 1. Their final formula, the **Marshall and Hoare Formula**, for $T_r(t) - T_a$ was

$$T_r(t) - T_a = \left(Ae^{Bt} + (1-A)e^{tAB/(A-1)}\right) \cdot (T_0 - T_a)$$

Remark: Notice that this formula, by moving T_a to the other side, can be written

$$T_r(t) = \left(Ae^{Bt} + (1-A)e^{tAB/(A-1)}\right) \cdot (T_0 - T_a) + T_a$$

Then, using the distributive law, this can be written as

$$T_r(t) = Ae^{Bt}(T_0 - T_a) + (1-A)e^{tAB/(A-1)}(T_0 - T_a) + T_a$$

And finally changing the order of the terms (i.e., using the commutative law), we can rewrite this as

$$T_r(t) = Ae^{Bt}(T_0 - T_a) + T_a + (1-A)e^{tAB/(A-1)}(T_0 - T_a)$$

Notice that if we leave off the complicated last term, we just have the formula for the temperature of an object according to *Newton's Law of Cooling*. The last term is what allows us to also account for the plateau.

In these formulas, B is a negative constant and A is a positive constant which are determined by taking temperature measurements while the body is cooling. However to determine the values of A and B exactly one must also know the environmental conditions, the size and fattiness of the body, and so on.

From the Marshall and Hoare Formula, what is $T_r(0) - T_a$? Try graphing $T_r(t) - T_a$ for $A = 1.55$, $B = -.05$, $T_0 = 37$ and $T_a = 15.5$ (note the temperatures are given in Celsius, but A and B have also been computed using Celsius temperatures, so you don't have to do any conversions here). Use a few different ranges, for example $0 \leq t \leq 5$, $0 \leq t \leq 20$, and $0 \leq t \leq 60$. Does your graph have the same general shape as the one in Figure 1? For how many hours (roughly) does your plateau last?

Computing A and B is also not that easy. To help coroners compute A and B for a particular case, Claus Henssge, Professor of Forensic Medicine at the University of Essen in Germany collected huge amounts of data, analyzed this data using statistical tools (such as regression analysis), and made a computer program that allows coroners to input data and get out an estimated time of death. Some tables of his data are attached in the following pages. In the first table, Table 3.3, he measured actual values for B for different bodies (with different body weights, bw) in moving air. These values are in the column labeled B_{wind} (there are other columns in the table as well, but we won't go into those now). In the second table, Table, 3.7, he measured actual B values for different bodies cooling in still water, B_{water}. And in the third table, Table 3.10, he measured actual B values for bodies cooling on different surfaces such as an armchair or a stone floor.

A simplified version of Henssge's program can be found at

`www.pathguy.com/TimeDead.htm.`

Let's use this to help us determine the time that Mr. Boddy (from *Project 1*) died. **Enter what you know into the program; ask your group lecturer any questions that you care to; and see if you can figure out who killed John Boddy.** Questions and discussion can be posted on your group's John Boddy discussion board and answers will be posted there as well.

Part II: A Little Statistics Never Killed Anyone

Professor Leonard Nokes of the Department of Medical Engineering at the University of Wales did a comparative analysis of eight different algorithms for computing time since death and he presented his results in a paper entitled *Analysis of Algorithms in Actual Cases*, the first two pages of which are in your packet (note that the article begins about 2/3 of the way down the first page). Read these two pages.

In Nokes' study, each body had really been dead for 10 hours, so a perfect algorithm would have given the answer of 10 every time. As you can see, there is no perfect algorithm, however certainly some did better than others.

We have already heard of the algorithms by Marshall-Hoare and Henssge. Let me explain two of the other algorithms that Nokes included in his comparison. A *Rule of Thumb* method is one which is easy and quick to use and gives a reasonable estimate of the answer most of the time. *Rule of Thumb A* is very old and is still often used today. It is based on the hypothesis that a dead body cools "on average" 1.5°F each hour (this average is taken over a long period of time and includes the plateau period). **What do you think of this hypothesis? Does it violate the way in which we have been thinking about the problem of cooling bodies. Come up with a formula for Rule of Thumb A and try it out on a few bodies from Table 2.3. Did you get the answers they got in the book?**

Rule of Thumb B is based on the hypothesis that after the plateau period, cooling occurs at approximately 1°C per hour. The plateau period is assumed to be three hours long (and it's assumed that the body has been dead for at least three hours). **Write a formula for Rule of Thumb B and try it out on a few bodies from Table 2.3.**

Which of the methods in the table do you think is the best and why? To answer this question, it might be nice to have a way to pictorially show all the errors. A picture would help you see which algorithm is most accurate overall and also help you convince me of your opinion. Use some of what you used in your stats class (how will you treat negative errors as opposed to positive ones?). Are you surprised by any of Nokes' data?

What are the circumstances under which the bodies cooled in the experiment? Do you think that the methods that performed better here will perform better in all situations? Why or why not? What are some experiments you might perform to explore the strengths and weaknesses of the different methods (I mean, supposing you had access to all the dead bodies you needed)?