Problem Set 3
Due: Thursday, February 10

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

1 Traditional Problems

1. (Gallian, Chapter 3 Exercises, #4) Prove that in any group, an element and its inverse have the same order.
2. (Gallian, Chapter 3 Exercises, #9) Show that if a is an element of a group G, then $|a| \leq |G|$.
3. (Gallian, Chapter 3 Exercises, #20) Let G be a group, and let $a \in G$. Prove that $C(a) = C(a^{-1})$.
4. (Gallian, Chapter 3 Exercises, #26) If H is a subgroup of G, then by the *centralizer* $C(H)$ of H we mean the set $\{x \in G \mid xh = hx \text{ for all } h \in H\}$. Prove that $C(H)$ is a subgroup of G.
5. (Gallian, Chapter 4 Exercises, #22) Prove that a group of order 3 must be cyclic.
6. (Gallian, Chapter 3 Exercises, #44) Suppose G is a group that has exactly eight elements of order 3. How many subgroups of order 3 does G have?

2 Computer Problems

As outlined on Problem Set 0, please intersperse your GAP commands and output with your explanations. You should create a log file as described in Chapter -1 of the lab manual. If you type up your solutions, you can cut and paste from this log file into your solution file; please use a different font so it is easy to tell what is what. If you hand-write your solutions, you should still print out your log file; then physically cut and paste it into your solutions.

1. (Lab Manual, Chapter 2 Exercises, #2.5) Read about the command `GL(n,p)` in Chapter 2 of the lab manual. Use this command to find the order of $\mathrm{GL}(2, \mathbb{Z}_p)$ and $\mathrm{SL}(2, \mathbb{Z}_p)$ for $p = 3, 5, 7$ and 11. What relationship do you see between the orders of $\mathrm{GL}(2, \mathbb{Z}_p)$ and $\mathrm{SL}(2, \mathbb{Z}_p)$ and $p - 1$? Does this relationship hold for $p = 2$? Based on these examples, does it appear that p always divides the order of $\mathrm{SL}(2, \mathbb{Z}_p)$? What about $p - 1$? What about $p + 1$? Guess a formula for the order of $\mathrm{SL}(2, \mathbb{Z}_p)$. Guess a formula for the order of $\mathrm{GL}(2, \mathbb{Z}_p)$.