Problem Set 5
Due: Monday, February 28

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

1   Traditional Problems

1. (Gallian, Chapter 5 Exercises, #8) What is the maximum order of any element in A_{10}?

2. (Gallian, Chapter 5 Exercises, #12) If α is even, prove that α^{-1} is even. If α is odd, prove that α^{-1} is odd.

3. (Gallian, Chapter 5 Exercises, #13) Prove Theorem 5.6. That is, prove that the set of even permutations in S_n forms a subgroup of S_n.

4. If G is an Abelian group and n is a fixed positive integer, then $G^n := \{g^n \mid g \in G\}$ is a subgroup of G. (You can assume this fact.)

    (a) Prove that if $G = S_n$, then $G^2 \subseteq A_n$.
    (b) Prove that if $G = S_n$ with $n = 4$ or $n = 5$, then $G^2 = A_n$.
    (c) Let $G = A_4$. Prove that G^2 is *not* a subgroup of G.

5. Let $\beta = (3,5)(2,3,5,4)(1,2,3,4) \in S_7$. Write β^{2009} as a product of disjoint cycles. Be sure to justify your answer. You must do this problem without the help of a computer.

2   Computer Problems

As outlined on Problem Set 0, please intersperse your GAP commands and output with your explanations. You should create a log file as described in Chapter -1 of the lab manual. If you type up your solutions, you can cut and paste from this log file into your solution file; please use a different font so it is easy to tell what is what. If you hand-write your solutions, you should still print out your log file; then physically cut and paste it into your solutions.

Do the following problems from Chapter 5 of the computer lab manual.

1. Problem 5.1 (You do not have to check your answers to parts (a) and (c) by hand.)

2. Problem 5.8

3. Problem 5.9

4. Problem 5.10