Problem Set 6
Due: Monday, March 7

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

1 Traditional Problems

1. (Gallian, Chapter 6 Exercises, #6) Prove that the notion of group isomorphisms is transitive. That is, if G, H, and K are groups and $G \approx H$ and $H \approx K$, then $G \approx K$.
2. (Gallian, Chapter 6 Exercises, #10) Let G be a group. Prove that the mapping $\alpha(g) = g^{-1}$ for all g in G is an automorphism if and only if G is Abelian.
3. (Gallian, Chapter 6 Exercises, #19) If ϕ and γ are isomorphisms from the cyclic group $\langle a \rangle$ to some group and $\phi(a) = \gamma(a)$, prove that $\phi = \gamma$.
4. (Gallian, Chapter 6 Exercises, #22) Prove Property 4 of Theorem 6.3: Suppose that ϕ is an isomorphism from a group G onto a group $\overline{G}$. If K is a subgroup of G, then prove that
$$\phi(K) = \{\phi(k) \mid k \in K\}$$
is a subgroup of $\overline{G}$.
5. (Gallian, Chapter 6 Exercises, #34) If a and g are elements of a group, prove that $C(a)$ is isomorphic to $C(gag^{-1})$.
6. Let G be a group. Prove that $|\text{Inn}(G)| = 1$ if and only if G is Abelian.