Problem Set 7
Due: Thursday, March 10

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

1   Traditional Problems

1. (Gallian, Chapter 7 Exercises, #9) Let $|a| = 30$. How many left cosets of $\langle a^4 \rangle$ in $\langle a \rangle$ are there? List them.

2. (Gallian, Chapter 7 Exercises, #14) Suppose that K is a proper subgroup of H and H is a proper subgroup of G. If $|K| = 42$ and $|G| = 420$, what are the possible orders of H?

3. (Gallian, Chapter 7 Exercises, #15) Let G be a group with $|G| = pq$, where p and q are prime. Prove that every proper subgroup of G is cyclic.

4. (Gallian, Chapter 7 Exercises, #20) Suppose H and K are subgroups of a group G. If $|H| = 12$ and $|K| = 35$, find $|H \cap K|$. Generalize.

5. (Gallian, Chapter 7 Exercises, #22) Suppose that H and K are subgroups of G and there are elements a and b in G such that $aH \subseteq bK$. Prove that $H \subseteq K$.

6. (Gallian, Chapter 7 Exercises, #31) Let H and K be subgroups of a finite group G with $H \subseteq K \subseteq G$. Prove that $|G : H| = |G : K||K : H|$.