

Problem Set 8

Due: 10:00 a.m. on Thursday, March 17

Work all of the following problems. Remember, you are encouraged to work together on Problem Sets, but each student must turn in his or her own write-up. Be sure to adhere to the Rules and Expectations outlined in the Course Information Sheet.

1 Traditional Problems

1. (Gallian, Chapter 7 Exercises, #26) Let G be a group of order 25. Prove that G is cyclic or $g^5 = e$ for all g in G .
2. (Gallian, Chapter 7 Exercises, modified #35) Let

$$G = \{(1), (12)(34), (1234)(56), (13)(24), (1432)(56), (56)(13), (14)(23), (24)(56)\}.$$

Find the stabilizer of 5 and the orbit of 5.

3. (Gallian, Chapter 7 Exercises, #36) Let G be a group of order p^n where p is prime. Prove that the center of G cannot have order p^{n-1} .
4. (Gallian, Chapter 7 Exercises, #38) Prove that a group of order 12 must have an element of order 2.
5. (Gallian, Chapter 7 Exercises, modified #48) Use the Orbit-Stabilizer Theorem to calculate the order of the group of rotations of a regular dodecahedron (a solid with 12 congruent pentagons as faces).

2 Computer Problems

As outlined on Problem Set 0, please intersperse your GAP commands and output with your explanations. You should create a log file as described in Chapter -1 of the lab manual. If you type up your solutions, you can cut and paste from this log file into your solution file; please use a different font so it is easy to tell what is what. If you hand-write your solutions, you should still print out your log file; then physically cut and paste it into your solutions.

Complete the following problems from Chapters 6 and 7 of the computer lab manual:

1. Problem 6.1.
2. Problem 7.1.
3. Problem 7.2.
4. Problem 7.3.
5. Problem 7.4.