Math 314/814–002: Matrix Theory **Dr. S. Cooper, Fall 2008**

Name: _____

Quiz 2 Solutions

Please write your solutions to the following exercises in the space provided. You should write legibly and fully explain your work.

Good Luck!

(1) Find the parametric equations of the plane in $\mathbb{R}^3$ passing through the point $P = (6, 0, 1)$ with direction vectors $\mathbf{u} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -3 \\ 3 \\ -1 \end{bmatrix}$. [6 pts]

Solution: The vector equation is

$$\mathbf{x} = \begin{bmatrix} 6 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} -3 \\ 3 \\ -1 \end{bmatrix}.$$

Thus, the parametric equations are

$$\begin{aligned} x &= 6 + 3s - 3t \\ y &= 3t \\ z &= 1 - s - t \end{aligned}$$

(2) Are the following equations linear? If not, then explain. [6 pts]

(a) $7 \cos x - 4y + z = \sqrt{3}$

Solution: This is not a linear equation because the variable x is the input of the function $\cos x$.

(b) $\cos(7)x - 4y + z = \sqrt{3}$

Solution: This is a linear equation.

(3) Find the augmented matrix of the linear system [5 pts]

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 1 \\ x_1 + x_3 &= -5 \\ -x_1 + 2x_2 - 2x_3 &= 0 \end{aligned}$$

Solution: $\left[\begin{array}{rrr|r} 2 & 3 & -1 & 1 \\ 1 & 0 & 1 & -5 \\ -1 & 2 & -2 & 0 \end{array}\right]$

(4) Let $\mathbf{v}, \mathbf{v_1}, \ldots, \mathbf{v_k}$ be vectors in $\mathbb{R}^n$. Define what it means for $\mathbf{v}$ to be a *linear combination* of $\mathbf{v_1}, \ldots, \mathbf{v_k}$. [3 pts]

Solution: $\mathbf{v}$ is a *linear combination* of $\mathbf{v_1}, \ldots, \mathbf{v_k}$ if there exist scalars $c_1, \ldots, c_k$ such that

$$\mathbf{v} = c_1\mathbf{v_1} + c_2\mathbf{v_2} + \cdots + c_k\mathbf{v_k}.$$