Name: ______________________________

Quiz 7 Solutions

Please write your solutions to the following exercises in the space provided. You should write legibly and fully explain your work.

Good Luck!

(1) Let $A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ -1 & -1 & 1 \end{bmatrix}$.

(a) *Using the definition of eigenvector*, show that $\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$ is an eigenvector of A. What is the eigenvalue of $\mathbf{x}$? [3 pts]

Solution: The vector $\mathbf{x}$ is non-zero and

$$A\mathbf{x} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0\mathbf{x}.$$

Since $A\mathbf{x} = 0\mathbf{x}$, $\mathbf{x}$ is an eigenvector of A with eigenvalue $\lambda = 0$.

(b) $\lambda = 2$ is an eigenvalue of A. Find a basis for the corresponding eigenspace E_2. What is the geometric multiplicity of the eigenvalue $\lambda = 2$? [8 pts]

Solution: To find a basis for E_2 we find the null space of $(A - 2I)$:

$$[A - 2I \mid \mathbf{0}] = \left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Thus,

$$E_2 = span\left(\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right).$$

A basis for E_2 is

$$\left\{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}.$$

Since $\dim(E_2) = 2$, the geometric multiplicity of $\lambda = 2$ is 2.

(2) Suppose that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$. What is $\begin{vmatrix} a & b & c \\ 4d-2g & 4e-2h & 4f-2i \\ g & h & i \end{vmatrix}$? Be sure to show all of your work. [4 pts]

Solution: Observe that we have the following row reductions:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \to B = \begin{bmatrix} a & b & c \\ 4d & 4e & 4f \\ g & h & i \end{bmatrix} \to C = \begin{bmatrix} a & b & c \\ 4d-2g & 4e-2h & 4f-2i \\ g & h & i \end{bmatrix}.$$

By Theorem 4.3,

$$\det(C) = \det(B) = 4\det(A) = 4(4) = 16.$$

(3) Suppose that A and B are $n \times n$ matrices with $\det(A) = -3$ and $\det(B) = 5$. Find $\det(B^{-1}A)$. Be sure to show all of your work. [5 pts]

Solution:

$$\det(B^{-1}A) = \det(B^{-1})\det(A) = \frac{1}{\det(B)}\det(A) = -\frac{3}{5}.$$