P2P Tools

Fractal Analysis – FA Tool

User's Manual

Version 1.50

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1. Semivariogram

The semivariogram $\gamma(h)$ for a 2D data set is given by (Chi et al. 2012):

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})}\sum_{i=1}^{N(h)}[Z(\mathbf{s}_i+\mathbf{h})-Z(\mathbf{s}_i)]^2 \quad (1)$$

where $\gamma(\mathbf{h})$ = semivariogram; $\mathbf{s}_i$ = location i; $\mathbf{h}$ = lag distance; $Z(\mathbf{s}_i)$ = elevation at location $\mathbf{s}_i$; $Z(\mathbf{s}_i+\mathbf{h})$ = elevation at location $(\mathbf{s}_i+\mathbf{h})$; and $N(\mathbf{h})$ = number of pairs spaced at $\mathbf{h}$. In practice, certain tolerance in terms of distance and direction is generally introduced in the computation of semivariogram.

2. Windows-based Fractal Analysis Tool

2.1 Computation of Semivariogram

The Windows-based fractal analysis (FA) program, developed using C#, facilitates the computations of semivariogram, fractal dimension (D), and intercept (I_c) (Chi et al. 2012). The program uses a polar system to calculate semivariograms. The origin of the coordinate system is located at the lower left corner. The east direction is defined as 0º and the angle increases along the counter-clockwise direction. Thus, 90º, 180º, and 270º represent north, west, and south directions, respectively. The FA tool is able to calculate directional semivariograms along any directions ranging from 0º to 360º. The omni-directional semivariogram can be calculated by setting the angle tolerance to a value greater than or equal to 90º. The basic parameters for calculating semivariograms include the number of lags, lag distance (h), lag tolerance (htol), angle (a), angle tolerance (atol), and bandwidth. The spatially-distributed input data can be either regular grid data (e.g., DEM) or scatter data. Note that computation of omni-directional semivariogram only requires lag distance and lag tolerance.

2.2 Computation of Fractal Dimension D

Based on the calculated semivariogram $\gamma(h)$, fractal dimension D and intercept Ic can be determined. For a fractal Brownian motion (fBm) model, the elevation change $\Delta Z(h)$ and the structural function are respectively given by (Huang and Bradford, 1992):

$$\Delta Z(h) \propto h^H \qquad (0 < H < 1) \quad (2)$$

and

$$\gamma(h) \propto h^{2H} \tag{3}$$

where H is the Hurst exponent, and $\Delta Z(h)$ is the difference in elevations at distance h. Thus, the semivariogram $\gamma(h)$ can be expressed as:

$$\gamma(h) = Kh^{2H} \tag{4}$$

or

$$\log[\gamma(h)] = 2H\log(h) + \log(K) \tag{5}$$

where K is the proportional factor. Eq. (5) shows a linear relationship between log[$\gamma(h)$] and log(h) with a slope (S) of $2H$ and an intercept (Ic) of log(K). That is, $H = S/2$. Given the Hurst exponent H, the fractal dimension D of a topographic surface (Euclidean dimension d = 3) is given by:

$$D = 3 - H = 3 - 0.5S \tag{6}$$

Following Huang and Bradford (1992), K in Eq. (4) can be expressed as a function of crossover length l:

$$K = l^{2-2H} \tag{7}$$

The crossover length l can be determined by K and H from the best-fit linear line of the log-log semivariogram curve. Fig. 1 schematically shows the procedures for determining D, Ic, and the breakpoint distance d_B by using the semivariogram method. The key step to calculate D is to find the best-fit linear segment of the semivariogram curve. The breakpoints are determined based on the distribution of $\gamma(h)$ and h on the log-log plot. The linear segment of the log-log semivariogram curve (Eq. 5) is then fitted by using the least-square regression method. The goodness of the least square regression is evaluated by:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(\gamma_i - \hat{\gamma}_i)^2}{\sum_{i=1}^{n}(\gamma_i - \bar{\gamma})^2} \tag{8}$$

where R^2 = coefficient of determination; γ_i = actual semivariogram at i^{th} distance; $\hat{\gamma}_i$ = estimated semivariogram at i^{th} distance; $\bar{\gamma}$ = average value of the actual semivariogram for all distances; and n = total number of lag distances h for semivariogram calculation.

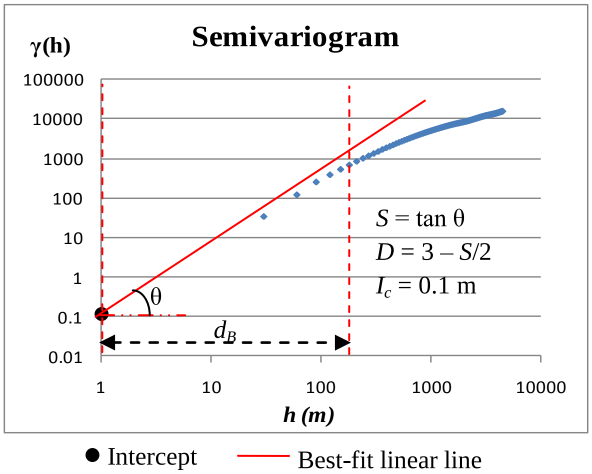


Fig. 1 Determination of fractal dimension D, ordinate intercept I_c, and breakpoint distance d_B

3. Testing of the FA Program

Performance of the FA program was evaluated by comparing with the GSLIB software package (Deutsh and Journel 1998). A series of surfaces with various topographic characteristics were selected for this purpose. Specifically, both omni-directional semivariogram and directional semivariogram (along any direction) were computed by using both software packages and the results were evaluated. Fig. 2 shows the comparisons of the omni-directional and directional (0º and 75º) semivariograms from the FA program and GSLIB for a selected land surface. Good agreement has been achieved.

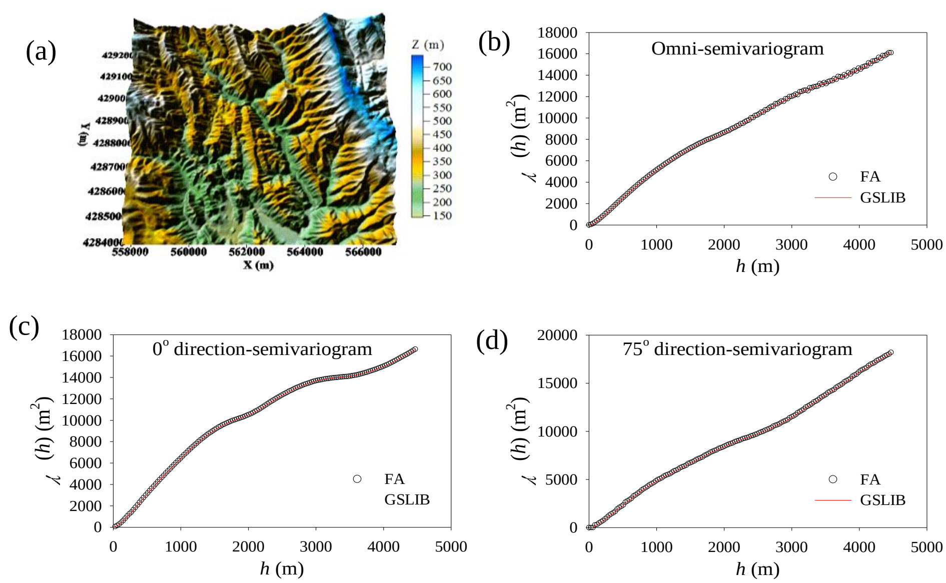


Fig. 2 Comparisons of the semivariograms computed by the FA program and GSLIB

4. Use of the FA Tool

The main Windows interface includes a map area and a control panel used for importing DEM data, inputting the required parameters, computing the semivariogram, plotting the semivariogram in a normal or log-log plot, and fitting the first linear segment of the semivariogram curve for computation of fractal dimension D and the ordinate intercept I_c (Fig. 3).

1) Load Data: Click the button "Input Data" and select the data file. The file can be in the format of ".dat" or ".txt". The data should have three columns in the order of x, y, and z. The data points can be either regularly or irregularly spaced.

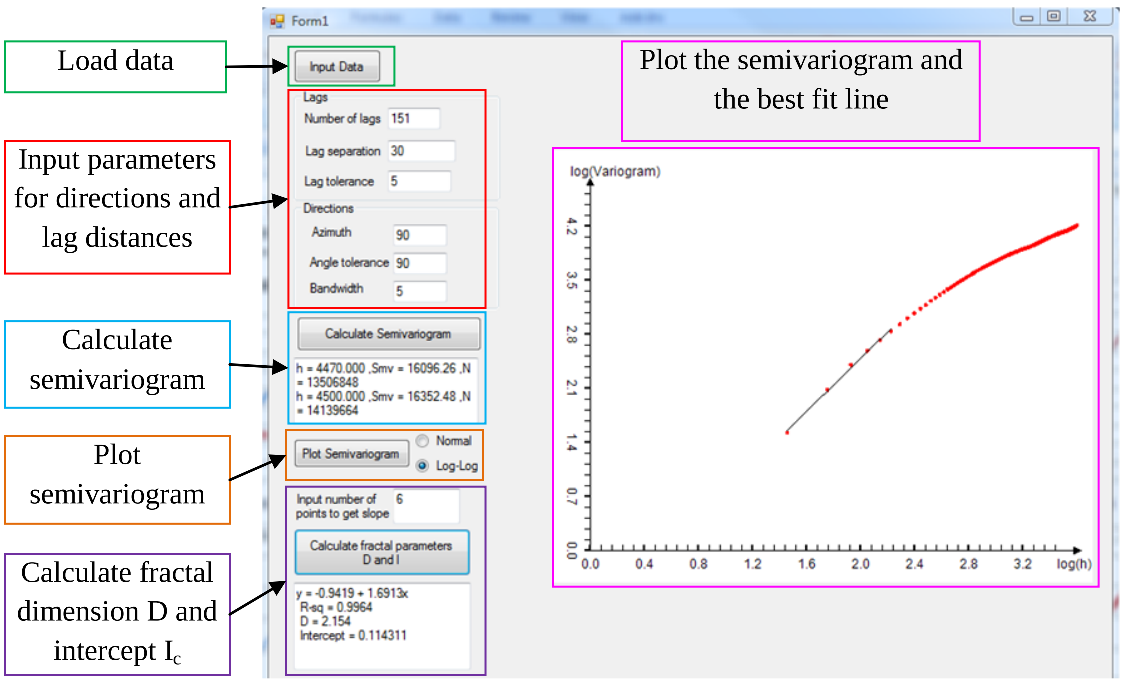
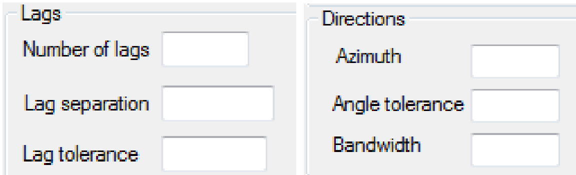
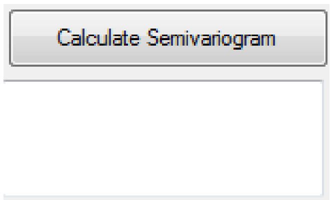


Fig. 3 Windows interface of the FA software

2) Input Parameters: There are two major groups of input parameters: lag and direction parameters. In the lag group, the required parameters include the number of lags, lag distance (h), and lag tolerance. In the direction group, the required parameters include angle/azimuth, angle tolerance, and bandwidth. Omni-directional semivariogram can be calculated by setting the angle tolerance greater than or equal to 90º.

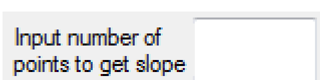


3) Calculate Semivariograms: After inputting all the parameters, click "Calculate Semivariogram." The calculation will then start and the results are shown in the following text box. The output results include the lag distances (*h*), *γ(h)*, and the number of pairs at the corresponding *h*.

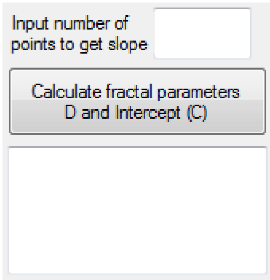


4) Plot Semivariograms: The calculated semivariogram can be plotted in either a normal plot or a log-log plot by checking the desired option.

5) Calculate Fractal *D* and Intercept (I_c): By fitting the linear line of the first linear segment of the semivariogram on the log-log plot, the slope of the linear line can be determined and is equivalent to fractal dimension *D*. From the log-log plot of the semivariogram distribution, users can define the number of points to obtain a best-fit linear line.



By clicking "Calculate fractal parameters *D* and Intercept (*Ic*)," the FA program will automatically fit the linear line for the selected points based on the least-square method. The final results include the fitted line (y = ax + b), R^2, *D*, and *Ic* values.



References

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Huang, C., and Bradford, J. M. (1992). “Application of a laser scanner to quantify soil microtopography.” *Soil Sci. Soc. of Am. J.*, 56(1), 14-21.