

## P2P Tools

## Coordinate Transformation XYT Tool

## User's Manual

## Version 1.50

Principal Investigator: Dr. Xuefeng Chu
Postdoctoral Research Associate: Dr. Jianli Zhang
Graduate Research Assistants: Jun Yang, Noah Habtezion, Yaping Chi, and Yingjie Yang


June 17, 2013

## Acknowledgement

This material is based upon work supported by the National Science Foundation under Grant No. EAR0907588. The P2P Tool software is a part of the NSF-funded project, titled "CAREER: MicrotopographyControlled Puddle-filling to Puddle-merging (P2P) Overland Flow Mechanism: Discontinuity, Variability, and Hierarchy." Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

## Contact Information for Limited Technical Support

Dr. Xuefeng Chu

Department of Civil Engineering (Dept 2470)
North Dakota State University
PO Box 6050, Fargo, ND 58108-6050
Tel.: 701-231-9758, Fax: 701-231-6185
E-mail: xuefeng.chu@ndsu.edu

A topographic surface can be rotated to generate surfaces with various slopes. The derived sloping surfaces benefit the investigation of the slope effects in hydrotopographic analyses. This coordinate transformation program (XYT) is designed to calculate the coordinates of the new sloping surfaces and obtain their DEMs. The computation of the new coordinates $(X, Y$, and $Z$ ) is based on the Euler equation (Arfken and Weber, 2005).

## 1. Methodology

The rotation process can be accomplished by multiplying the DEM matrix of the original surface by rotational matrices for rotations about the $X, Y$, and $Z$ axes. These three rotational matrices are composed of $\sin$ and $\cos$ functions of the user-specified rotation angles and can be respectively given by (Arfken and Weber, 2005):

$$
\begin{align*}
& \mathbf{R}_{\mathrm{x}}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \beta_{1} & \sin \beta_{1} \\
0 & -\sin \beta_{1} & \cos \beta_{1}
\end{array}\right]  \tag{1}\\
& \mathbf{R}_{\mathrm{Y}}=\left[\begin{array}{ccc}
\cos \beta_{2} & 0 & \sin \beta_{2} \\
0 & 1 & 0 \\
-\sin \beta_{2} & 0 & \cos \beta_{2}
\end{array}\right]  \tag{2}\\
& \mathbf{R}_{\mathrm{Z}}=\left[\begin{array}{ccc}
\cos \beta_{3} & \sin \beta_{3} & 0 \\
-\sin \beta_{3} & \cos \beta_{3} & 0 \\
0 & 0 & 1
\end{array}\right] \tag{3}
\end{align*}
$$

where $\mathbf{R}_{\mathbf{x}}, \mathbf{R}_{\mathbf{y}}$, and $\mathbf{R}_{\mathbf{z}}$ are the rotational matrices for rotations about the $X, Y$, and $Z$ axes, respectively; and $\beta_{1}, \beta_{2}$, and $\beta_{3}$ are the rotation angles along the $X, Y$, and $Z$ axes, respectively. In the program, any sloping surface is generated by rotating the original surface sequentially along the $X, Y$, and $Z$ axes with angles of $\beta_{1}, \beta_{2}$, and $\beta_{3}$, respectively. The DEM matrix of the final rotated surface can be expressed as:

$$
\begin{equation*}
\mathbf{P}^{\prime \prime \prime}=\mathbf{P R}_{x} \mathbf{R}_{y} \mathbf{R}_{z}=\mathbf{P}^{\prime} \mathbf{R}_{y} \mathbf{R}_{z}=\mathbf{P}^{\prime \prime} \mathbf{R}_{z} \tag{4}
\end{equation*}
$$

in which

$$
\begin{align*}
& \mathbf{P}^{\prime}=\left[\begin{array}{lll}
x_{1}^{\prime} & y_{1}^{\prime} & z_{1}^{\prime} \\
x_{2}^{\prime} & y_{2}^{\prime} & z_{2}^{\prime} \\
\ldots & \ldots & \ldots \\
x_{n}^{\prime} & y_{n}^{\prime} & z_{n}^{\prime}
\end{array}\right]=\mathbf{P R}_{x}=\left[\begin{array}{lll}
x_{1} & y_{1} & z_{1} \\
x_{2} & y_{2} & z_{2} \\
\ldots & \ldots & \ldots \\
x_{n} & y_{n} & z_{n}
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \beta_{1} & \sin \beta_{1} \\
0 & -\sin \beta_{1} & \cos \beta_{1}
\end{array}\right]  \tag{5}\\
& \mathbf{P}^{\prime \prime}=\left[\begin{array}{lll}
x_{1}^{\prime \prime} & y_{1}^{\prime \prime} & z_{1}^{\prime \prime} \\
x_{2}^{\prime \prime} & y_{2}^{\prime \prime} & z_{2}^{\prime \prime} \\
\ldots & \ldots & \ldots \\
x_{n}^{\prime \prime} & y_{n}^{\prime \prime} & z_{n}^{\prime \prime}
\end{array}\right]=\mathbf{P}^{\prime} \mathbf{R}_{y}=\left[\begin{array}{lll}
x_{1}^{\prime} & y_{1}^{\prime} & z_{1}^{\prime} \\
x_{2}^{\prime} & y_{2}^{\prime} & z_{2}^{\prime} \\
\ldots & \ldots & \ldots \\
x_{n}^{\prime} & y_{n}^{\prime} & z_{n}^{\prime}
\end{array}\right]\left[\begin{array}{ccc}
\cos \beta_{2} & 0 & \sin \beta_{2} \\
0 & 1 & 0 \\
-\sin \beta_{2} & 0 & \cos \beta_{2}
\end{array}\right]  \tag{6}\\
& \mathbf{P}^{\prime \prime \prime}=\left[\begin{array}{lll}
x_{1}^{\prime \prime \prime} & y_{1}^{\prime \prime \prime} & z_{1}^{\prime \prime \prime} \\
x_{2}^{\prime \prime \prime} & y_{2}^{\prime \prime \prime} & z_{2}^{\prime \prime \prime} \\
\ldots & \cdots & \ldots \\
x_{n}^{\prime \prime \prime} & y_{n}^{\prime \prime \prime} & z_{n}^{\prime \prime \prime}
\end{array}\right]=\mathbf{P}^{\prime \prime} \mathbf{R}_{z}=\left[\begin{array}{lll}
x_{1}^{\prime \prime} & y_{1}^{\prime \prime} & z_{1}^{\prime \prime} \\
x_{2}^{\prime \prime} & y_{2}^{\prime \prime} & z_{2}^{\prime \prime} \\
\cdots & \cdots & \ldots \\
x_{n}^{\prime \prime} & y_{n}^{\prime \prime} & z_{n}^{\prime \prime}
\end{array}\right]\left[\begin{array}{ccc}
\cos \beta_{3} & \sin \beta_{3} & 0 \\
-\sin \beta_{3} & \cos \beta_{3} & 0 \\
0 & 0 & 1
\end{array}\right] \tag{7}
\end{align*}
$$

where $\mathbf{P}$ is the original DEM matrix; and $\mathbf{P}^{\prime}, \mathbf{P}^{\prime \prime}$, and $\mathbf{P}^{\prime \prime \prime}$ are the DEM matrices after sequential rotations about the $X, Y$ and $Z$ axes with angles of $\beta_{1}, \beta_{2}$, and $\beta_{3}$, respectively.

## 2. Windows Interface



## 3. Procedures

1) Input data: By clicking the button "Input Data," users can load the original surface DEM data. The data requires three columns in the order of $\mathrm{X}, \mathrm{Y}$, and Z .
2) Rotation parameters: The original surface can be rotated based on either an angle or a slope, which can be selected by checking the option button "Angles" or "Slopes (\%)." Then, users need to input the desired angles or slopes along the $\mathrm{X}, \mathrm{Y}$, and Z axes. Note that the angles or slopes can be zero.
3) Rotation: Click the button "Rotate" to execute the surface coordinate transformation program.
4) Results: The new transformed data can be saved to a file in a user specified directory by clicking the button "Save As."

## Reference:

Arfken, G. B., and Weber, H. J. (2005). Mathematical Methods for Physicists. Elsevier Academic Press, San Diego. p199-203.

